4/6/16 "Make sure to prepare to do well"-Mr. Callahan

HW: Finish Review Sheet
Quarter Test on Friday

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Name: _____

Review for quarter 3 exam. Please not this review is not comprehensive. You should study all class notes and homework assignments.

1) If $\log_9 x = \frac{3}{2}$, what is the value of x?

A) $\frac{3}{2}$ B) 8 C) 27 D) $\frac{27}{2}$

$9^{\frac{3}{2}} = x$

2) If $\log_x 9 = \frac{1}{2}$ what is the value of x?

A) 27 B) 81 C) 3 D) $4\frac{1}{2}$

$x^{\frac{1}{2}\left(\frac{2}{1}\right)} = 9^{\frac{2}{1}}$ $x = 81$

3) The graph of $y = \log x$ lies in which quadrants?

A) I and II B) II and III C) I and IV D) III and IV

4) The expression $2\log_5 m + \log_5 n$ is equivalent to

A) $\log_5 (\frac{2m}{n})$ B) $\log_5 (\frac{m^2}{n})$ C) $\log_5 \sqrt{mn}$ D) $\log_5 m^2 n$

$2\log_5 m + \log_5 n$

$\log_5 m^2 + \log_5 n$

$\log_5 m^2 n$

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5) If $\log 5 = a$, then $\log 250$ can be expressed as

A) $2a + 1$ B) $50a$ C) $25a$ D) $10 + 2a$

$\log 250 = \log(5 \cdot 50) = \log(5 \cdot 5 \cdot 10)$

$\log 5 + \log 5 + \log 10 = a + a + 1$

6) The growth of a colony of cells can be determined by the formula $G = I(3.1)^{0.226t}$, in which G represents the final number in the colony, I is the initial number of cells, and t represents elapsed time in hours. Find how many hours it will take for a colony starting at 25 cells to increase to a size of 25,000 cells. [*Round the answer to the nearest whole hour.*] [*Show all work.*]

$G = 25000$

$I = 25$

$t = ?$

$\frac{25000}{25} = \frac{25(3.1)^{.226t}}{25}$

$1000 = (3.1)^{.226t}$

$\log_{3.1} 1000 = .226t$

$\frac{\log_{3.1} 1000}{.226} = \frac{.226t}{.226}$

$t = 27$ hrs

7) It has been shown that homes in a certain city increase in value at a rate of 7.5% per year. The value V of a home after t years is given by the formula $V = C(1 + r)^t$ where r is the rate of appreciation. If a home costs $42,000 in 2001, by what year will this home have doubled in value? [*Show all work.*]

$V = 2(42000) = 84000$

$C = 42000$

$r = .075$

$\frac{84000}{42000} = \frac{42000(1+.075)^t}{42000}$

$2 = (1.075)^t$

$\log_{1.075} 2 = t$

$t = 9.584$

Starting in 2001

$2001 + 9.584$

2010.584

Therefore it doubles sometime in 2010

8) Sean invests $10,000 at an annual rate of 5% compounded continuously, according to the formula $A = Pe^{rt}$, where A is the amount, P is the principal, $e = 2.718$, r is the rate of interest, and t is time, in years.

(a) Determine, to the nearest dollar, the amount of money he will have after 2 years. [*Show all work.*]

(b) Determine how many years, to the nearest year, it will take for his initial investment to double. [*Show all work.*]

9) A ball is dropped from a height of 8 feet and allowed to bounce. Each time the ball bounces, it bounces back to half its previous height. The vertical distance the ball travels, d, is given by the formula $d = 8 + 16\sum_{k=1}^{n}(\frac{1}{2})^k$, where n is the number of bounces. Based on this formula, what is the total vertical distance that the ball has traveled after four bounces? $n = 4$

A) 23.0 ft B) 22.0 ft C) 15.0 ft D) 8.9 ft

10) Find the value of $\sum_{k=2}^{5}(k-1)^3$.

$(2-1)^3 + (3-1)^3 + (4-1)^3 + (5-1)^3$

$1 + 8 + 27 + 64$

100

8) a) $P = 10000$
$r = .05$
$t = 2$

$A = 10000(2.718)^{.05(2)}$

$A \approx \$11052$

b) $A = 2(10000) = 20000$
$P = 10000$
$r = .05$
$t = ?$

$\frac{20000}{10000} = \frac{10000(2.718)^{.05t}}{10000}$

$2 = 2.718^{.05t}$

$\frac{\log_{2.718} 2}{.05} = \frac{.05t}{.05}$

$t = 14$ years

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11) What is the third term of the sequence defined by $t_1 = \frac{1}{3}$ and $t_n = 3t_{n-1} + 1$ when $n > 1$?

A) 9 B) 5 C) 7 D) 11

$t_1 = \frac{1}{3}$ $t_2 = 3(\frac{1}{3}) + 1$ $t_2 = 2$ $t_3 = 3(2) + 1 = 7$

12) What is t_{27} of the sequence 15, 10, 5, ...?

A) -115 B) -75 C) $\frac{1}{625}$ D) 0

$d = -5$

$t_n = t_1 + d(n-1)$

$t_{27} = 15 + -5(27-1) = -115$

13) What is the sum of the series 3 + 12 + 21 + ... + 597?

A) 19,200 B) 19,500 C) 19,800 D) 20,100

14) Find the common ratio of the sequence 8, -2, $\frac{1}{2}$, $-\frac{1}{8}$, ...?

A) -4 B) -8 C) $-\frac{1}{8}$ D) $-\frac{1}{4}$

$\frac{-2}{8} = -\frac{1}{4}$

15) What is t_8 of the geometric sequence 6,561, -2,187, 729, -243, ...?

A) -1 B) -3 C) 1 D) -9

$\frac{-2187}{6561} = -\frac{1}{3} = r$

recall $a_n = a_1 \cdot r^{n-1}$

$t_8 = 6561\left(-\frac{1}{3}\right)^{8-1}$

$t_8 = -3$

13) Need to Know "n" $d=9$

$$Sum = \frac{n}{2}(a_1 + a_n)$$

$$Sum = \frac{67}{2}(3+597)$$

$$Sum = 20100$$

(D)

$$a_n = a_1 + d(n-1)$$

$$597 = 3 + 9(n-1)$$

$$-3 \quad -3$$

$$\frac{594}{9} = \frac{9(n-1)}{9}$$

$$66 = n-1$$

$$+1 \quad +1$$

$$67 = n$$

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16) Find the sixth term of the geometric sequence when $a_4 = -8$ and $r = 0.5$.

A) -2 B) 2 C) -4 D) 4

$-8 \xrightarrow{\times .5} -4 \xrightarrow{\times .5} -2$

$a_1, a_2, a_3, a_4, a_5, a_6$

17) During a flu outbreak, a hospital recorded 8 cases the first week, 20 cases the second week, and 50 cases the third week.

(a) Write a geometric sequence to model the flu outbreak.

$r = \frac{20}{8} = 2.5$ $\quad a_n = 8(2.5)^{n-1}$

(b) If nothing could be done to prevent the spread of the flu, in which week would the total number of infected people exceed 12,000? [*Show all work.*]

$\frac{12000}{8} = \frac{8(2.5)^{n-1}}{8}$ $\quad a_n$

$1500 = 2.5^{n-1}$

$\log_{2.5} 1500 = n-1$

$\log_{2.5}(1500) + 1 = n$ $\quad +1$

$n = 8.98$

During the 8th week

18) The table below shows the December balance in a fixed-rate compound saving account each year from 1996 to 2000.

Multiplicative by 1.1 every time

Year	1996	1997	1998	1999	2000
Balance	$20,000	$22,000	$24,200	$26,620	$29,282

$r = 1.1$

What is the sum of the December balances from 1996 to 2006, inclusive? [*Show all work.*]

$\times 1.1 \quad \times 1.1$

$n = 1$

$\text{Sum} = \frac{a_1(1-r^n)}{1-r} = \frac{20000(1-1.1^{11})}{1-1.1} = 370{,}623.34$

19) Solve for x: $\log_2(x-3) + \log_2(x+1) = 5$

A) {-7, 5} B) {-5, 7} C) 5, only D) 7, only

$\log_2(x-3) + \log_2(x+1) = 5$

$\log_2(x-3)(x+1) = 5$

$\log_2 x^2 - 2x - 3 = 5$

Switch to exponential form

$2^5 = x^2 - 2x - 3$

$32 = x^2 - 2x - 3$

$-32 \qquad -32$

$0 = x^2 - 2x - 35$

$(x-7)(x+5)$

$x = 7 \quad | \quad x = -5$

reject b/c it makes log of negative

20) If $\log(x-3)+\log(x+4)-\log x=\log 5$, then the solution set for x is

A) {2, 6} B) {6} C) {-6, 2} D) {-2, 6}

21) Which of the following angles is coterminal with 915°?

A) 15° B) 195° C) 105° D) 75°

$915 - 360 = 555$ $\quad 555 - 360 = 195$ ✓

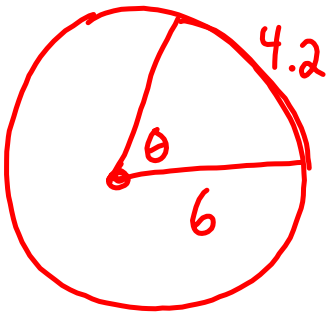
22) What is 600° converted to radians in terms of π?

A) $\frac{10\pi}{6}$ B) $\frac{5\pi}{3}$ C) $\frac{11\pi}{6}$ D) $\frac{10\pi}{3}$

$600 \cdot \frac{\pi}{180} = \frac{600\pi}{180} = \frac{10\pi}{3}$

23) A wedge-shaped piece is cut from a circular pizza. The radius of the pizza is 6 inches. The rounded edge of the crust of the piece measures 4.2 inches. To the nearest tenth, the angle of the pointed end of the piece of pizza, in radians, is

A) 0.7 B) 7.0 C) 25.2 D) 1.4



$\theta = \frac{s}{r}$

$\theta = \frac{4.2}{6}$

$\theta = .7$

20) $\log(x-3) + \log(x+4) - \log x = \log 5$

$\log \frac{(x-3)(x+4)}{x} = \log 5$

$\frac{x^2+x-12}{x} = \frac{5}{1}$

$x^2+x-12 = 5x$

$-5x \quad -5x$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6 \quad x=-2$

↑ makes log of negative

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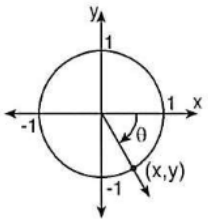
24) In the accompanying diagram of a unit circle, the ordered pair (x,y) represents the point where the terminal side of θ intersects the unit circle.

$-\frac{\pi}{3} = -60°$

$RA = 60°$

Y is the sin



Q: IV

R: 60

S: negative

T: $-\sin 60 = -\frac{\sqrt{3}}{2}$

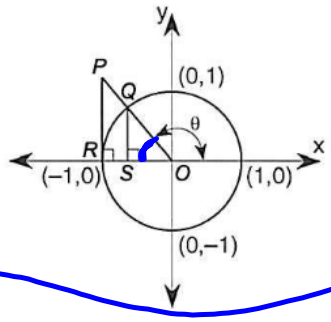
If $\theta = -\frac{\pi}{3}$, what is the value of y?

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $-\frac{\sqrt{2}}{2}$ D) $-\sqrt{3}$

25) In the accompanying diagram of a unit circle, $\overline{PR}$ is tangent to circle O at R, $\overline{QS} \perp \overline{OR}$, and $\overline{PR} \perp \overline{OR}$.

$\sin = \frac{opp}{hyp}$

$\sin = \frac{\overline{QS}}{\overline{QO}}$



Since its a unit circle

$\overline{QO} = 1$

$\therefore \sin = \frac{\overline{QS}}{1}$

Which of the following measures represents sin θ?

A) $\overline{PR}$ B) $\overline{SO}$ C) $\overline{QS}$ D) $\overline{RO}$

$\sin < 0$

sin is (−)

QIII

QIV

$\tan = -\frac{4}{5}$

tan (−)

QII

QIV

S	A
T	C

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negative

26) If $\sin x = -\frac{2}{3}$ and $\sin x \cos x > 0$, in which quadrant does angle x lie?

A) *I* B) *II* C) *III* D) *IV*

(-)(-)

Sin	ALL
tan	cos

27) If $\cos\theta = -\frac{3}{4}$ and $\tan\theta$ is negative, what is the value of $\sin\theta$?

A) $\frac{7}{4}$ B) $\frac{\sqrt{7}}{4}$ C) $\frac{4}{5}$ D) $-\frac{\sqrt{7}}{4}$

$\sin^2 + \cos^2 = 1$

$\sin^2 + \left(\frac{-3}{4}\right)^2 = 1$

$\sin^2 + \frac{9}{16} = 1$

$-\frac{9}{16} \quad -\frac{9}{16}$

$\sin^2 = \frac{7}{16}$

$\sin = \pm\frac{\sqrt{7}}{4}$

cos = −
tan = −
in QII
∴ $\sin = \frac{\sqrt{7}}{4}$

28) The multiplicative inverse of $\sin^2(90°) + \cos^2(90°)$ equals

A) 0 B) undefined C) -1 D) 1

$\sin^2(90) = 1^2 = 1$

$\cos^2(90) = 0^2 = 0$

$1 + 0 = 1$

multiplicative inverse of 1 is 1

29) The value of $\sin\left(\frac{3\pi}{2}\right) - \cos\left(\frac{\pi}{3}\right)$ is

A) $\frac{1}{2}$ B) $1\frac{1}{2}$ C) $-1\frac{1}{2}$ D) $-\frac{1}{2}$

$\frac{3\pi}{2} = \frac{3(180)}{2} = 270$

$\frac{\pi}{3} = \frac{180}{3} = 60$

$\sin(270) - \cos(60)$

$-1 - \frac{1}{2} = -1\frac{1}{2}$

30) If $f(x) = \cos 2x$, find the numerical value $f\left(\frac{\pi}{2}\right)$.

$\frac{\pi}{2} = 90$

$f(90) = \cos(2(90))$

$= \cos(180)$

$= -1$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2(90) + \cos^2(90) = 1$$

31) Evaluate: $\sin 90° - \cos 270°$

$1 - 0$

1

32) Which of the following is the reference angle of 280°?

A) -10° B) 80° C) -80° D) 100°

280 in QIV

$360 - 280 = 80$

33) The value of $\sin(-210°)$ is

A) $-\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$

$-210 + 360 = 150$

Q: II

R: $180 - 150 = 30$

S: positive

T: $\sin 30 = \frac{1}{2}$

34) The roots of the equation $x^2 + 6x + 11 = 0$ are

A) real, rational, and equal C) real, rational, and unequal

B) imaginary D) real, irrational, and unequal

$b^2 - 4ac$ $6^2 - 4(1)(11)$

$36 - 44 = -8$ ← negative means imaginary

35) What is the radius of a circle having the equation $x^2 + y^2 - 4x + 6y - 12 = 0$?

A) 16 B) 4 C) 5 D) 3.2

$+12 + 12$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$

25

Half Square Share

$\frac{-4}{2} = -2$ $(-2)^2 = 4$

$\frac{6}{2} = 3$ $3^2 = 9$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 25$

$(x-2)^2 + (y+3)^2 = 5^2$

Center = $(2, -3)$

radius = 5